

Recent Developments in Switching Theory

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Recent Developments in Switching Theory

**Edited by
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PREFACE

During the last decade a number of basic and significant advances have been made in switching theory. Some of these have their roots in classical switching theory while others are of recent origin, primarily from the impact of large-scale-integrated semiconductor technology in digital circuit design. To keep abreast of the recent developments in switching theory and to help maintain a sense of unity in this area, there is a need for presenting these results in a comprehensive book.

A summary of topics included in this book is as follows: Chapters I–IV deal with classical switching theory problems. Chapter I develops a new and simplified proof of Post's theorem on completeness of logic primitives and presents some recent results on simple bases and almost complete sets of logic primitives. Chapter II provides a deep insight into the role of feedback in combinational switching circuits and proves the "minimal-NOT" result for circuits with feedback. Chapter III presents a systematic procedure for the design of Lupanov decoding networks and gives the best results known so far on this subject. Chapter IV gives a review of classical results on counting theorems and their application to the classification of switching functions under different notions of equivalence, including linear and affine equivalences.

Chapter V is a self-contained development of abstract harmonic analysis of combinational switching functions. This chapter adds new conceptual insights and unifying principles to the traditional approaches to switching theory and also provides synthesis algorithms suitable for the new technology.

Chapters VI–IX deal with problems whose motivations primarily come from large-scale-integration technology. Chapter VI presents the theory of universal logic modules, methods of their construction, and upper bounds on the input terminals. Chapter VII is concerned with cellular logic: theory of single-rail cascades, elementary theory of two-rail cascades, and logical design techniques for combinational functions using two-dimensional cellular arrays. A discussion of minimization problems in cellular arrays and a review of research in the macrocellular area are also included. Chapter VIII is concerned with systematic techniques for the realization of multi-output logic functions

by means of multirail cellular cascades. A mathematical theory of multirail cascades—the decomposition theory of group functions—is developed which permits systematic techniques of great generality to be applied to the logic synthesis problem. Chapter IX presents programmable cellular logic, a form of versatile cellular logic whose behavior can be electronically programmed. The design and operation of several types of programmable cellular arrays for the realization of arbitrary logical behavior are described. Four special-purpose cellular arrays are offered to illustrate the theoretical techniques employed in the logical design of programmable arrays.

This book will be of general interest to all English-reading research workers in the major fields of computer science and discrete information processing systems. The book can also be used as a textbook for a graduate course on switching theory in computer science and electrical engineering as well as a supplementary reference book to already-established courses in switching theory in the universities. As far as possible, each chapter has been made self-contained, but a little background in modern algebra and classical switching theory will be helpful.

We believe that this book will create new interest in switching theory research and will provide stimulation for organizing new switching theory courses in universities.

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I would like to dedicate my contributions in this book to the memory of my late father, Surendra Nath Mukhopadhyay, and to my affectionate mother. I thank my wife, Pampa, for her understanding, and my daughters, Mita and Paula, for their love.



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